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Maximum *Q*-Factor of Microstrip Resonators

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***Abstract*—The quality factors of microstrip half-wavelength resonators have been calculated as a function of substrate thickness, for frequencies in the range 8–96 GHz, for different ϵ_r. Conductor, dielectric, and radiation losses have been included. The optimum substrate thickness for the maximum *Q*-factor for 50-Ω microstrip resonators has been derived as a function of frequency for different dielectric constants.**

I. Introduction

MICROSTRIP is used as the guiding and interconnecting structure in microwave integrated circuit modules up to 100 GHz. While microstrip conductor and dielectric dissipation may be expressed as loss per unit length, radiation losses may only be related to specific lengths of line. The quality factors of resonators, which include radiation losses, thus provide a guide to the losses that occur in microstrip. The *Q*-factors of half-wavelength resonators have been calculated as functions of substrate thickness and frequency, for some commonly used dielectric material. Belohoubek and Denlinger [1] have previously reported *Q*-factor calculations for $\lambda/4$ resonators using Pucel, Masse, and Hartwig's method [2] for conductor loss and Lewin's method [3] for radiation loss, over a limited frequency range. Our calculations show good agreement with their results for the 8-GHz case.

From these results, the optimum substrate thickness for maximum *Q*-factors of half-wavelength resonators may be obtained for a given frequency. In cases where radiation losses and spurious coupling problems exist, these calculations provide a guide to the optimum substrate thickness. When packaging eliminates these effects, the thickness is limited by the spurious mode excitation in the structure. Choice of substrate thickness may also be determined by thermal conductance, circuit, and processing considerations as well as material availability.

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II. Theoretical Considerations

The stored energy U, in a $\lambda_g/2$ resonator with a voltage distribution of $V \sin \beta_g z$ is given by

$$U = \frac{V^2}{8 Z_0 f} \tag{1}$$

where Z_0 is the resonator impedance and f is the frequency. The conductor and dielectric losses W_l in the resonator are given by

$$W_l = \frac{1}{4} \frac{V^2}{Z_0} \lambda_g (\alpha_d + \alpha_c) \tag{2}$$

where α_c and α_d are the conductor and dielectric loss constants in nepers per unit length. Thus the circuit quality factor Q_0 is given by

$$Q_0 = \frac{2\pi f U}{W_l} = \frac{\pi}{\lambda_g(\alpha_c + \alpha_d)}. \tag{3}$$

The radiation Q is estimated by calculating the total power radiated W_r and evaluating the ratio

$$Q_r = \frac{2\pi f U}{W_r}. \tag{4}$$

The total quality factor Q_t is given by

$$\frac{1}{Q_t} = \frac{1}{Q_0} + \frac{1}{Q_r}. \tag{5}$$

The conductor loss constant α_c is estimated from the expression [2]

$$\alpha_c = \frac{R_s}{2 Z_0 I^2} \left[\int_{-w/2}^{+w/2} J_s^2 \, dx + \int_{-\infty}^{+\infty} J_{gp}^2 \, dx \right] \tag{6}$$

where

J_s strip current;
J_{gp} ground plane current;
I total strip or ground plane current;

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R_s surface resistivity of the metalization;
Z_0 microstrip impedance.

It is usual to perform microstrip calculations with the loss normalized to the substrate height and in this case the appropriate loss constant is obtained from

$$\alpha_c = \frac{R_s}{2Z_0 h} F\left(\frac{w}{h}\right) \mathrm{Np/m} \tag{7}$$

where

$$F\left(\frac{w}{h}\right) = \frac{1}{I^2}\left[\int_{-w/2}^{w/2} J_s^2\, dx + \int_{-\infty}^{+\infty} J_{gp}^2\, dx\right].$$

The currents J_s and J_{gp} are the longitudinal currents and obtained from the inductance calculation. Since R_s varies as $\sqrt{f}$, thus α_c varies linearly as $\sqrt{f}$ and inversely as h.

Since most substrate materials used in microwave integrated circuits have low loss, a detailed estimate of dielectric losses is not necessary, and the plane wave approach due to Welch and Pratt [4] may be adopted

$$\alpha_d = \frac{\pi q \epsilon_r \tan\delta}{\epsilon_{\mathrm{eff}} \lambda_g} \mathrm{Np/m} \tag{8}$$

where

$$q = \frac{\epsilon_{\mathrm{eff}} - 1}{\epsilon_r - 1}$$

ϵ_{eff} effective dielectric constant;
ϵ_r substrate relative dielectric constant;
λ_g guide wavelength
$\tan\delta$ substrate loss tangent.

The radiated power is estimated by assuming that the sources of radiation include z-directed current on the resonator distributed as $I_m \cos\beta_g z$, its image in the ground plane and a polarization current directed normal to the ground plane (y-directed) and given by

$$\bar{i}_y dz = -2h\left(\frac{\epsilon_{r-1}}{\epsilon_r}\right) I_m \beta_g \sin\beta_g z\, dz. \tag{9}$$

The radiated power is found by integration of the appropriate expressions, [5], [6] from which Q_r is obtained. The calculations in this paper were performed using numerical integrations. Note that Lewin's method [3] would assume that the radiation from the two ends of the resonator are independent, which is not strictly valid for the half-wavelength microstrip resonator, where the length of the resonator may be small compared to the free space wavelength. Hence, the method of Easter and Roberts [5] seems to be appropriate in the present case.

Easter and Roberts [5] also give an approximate expression for the radiation quality factor

$$Q_r = \frac{3\epsilon_{\mathrm{eff}} Z_0 \lambda_0^2}{32\eta h^2} \tag{10}$$

which is valid for large ϵ_r, when $(\epsilon_r - 1)/\epsilon_r \simeq 1.0$.

The quasi-static parameters of the microstrip line were calculated using the method of images. The strip is assumed to be thick and hence the currents on the top and bottom surfaces of the strip obtained from this calculation are used for the evaluation of α_c. Dispersion is corrected using Getsinger's formula [7], the velocity is estimated using the corrected value of ϵ_{eff} with which the impedance is calculated. Values of α_d, Q_0, and Q_r and Q_t are then calculated, and the results obtained are discussed below.

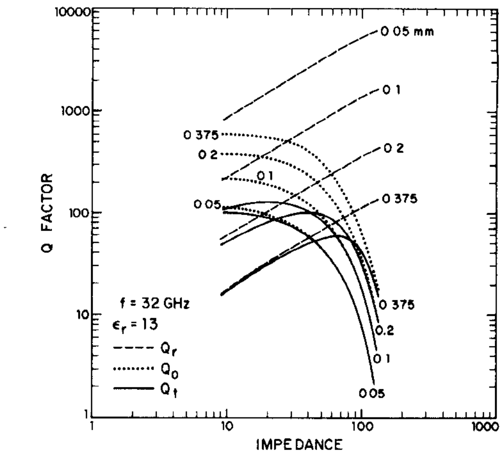

Fig. 1. Q-factor of $\lambda/2$ resonators against impedance for different thickness substrates. $\epsilon_r = 13$, $\tan\delta = 1\times10^{-4}$, $f = 32$ GHz, strip metal gold 3 μm thick.

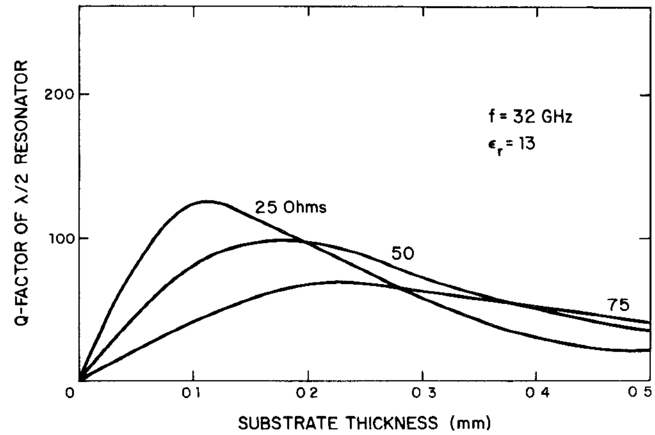

Fig. 2. Q-factor of $\lambda/2$ against substrate thickness for line impedances of 25, 50, and 75 Ω. $\epsilon_r = 13$, $\tan\delta = 10^{-4}$, $f = 32$ GHz.

III. Results

A typical set of curves obtained from the calculations are shown in Fig. 1, where the resonator Q-factor is plotted against microstrip impedance for different substrate thickness. The particular set shown in Fig. 1 is for a GaAs substrate at 32 GHz, with $\epsilon_r = 13$, $\tan\delta = 6\times10^{-4}$, and the strip thickness $= 3\mu$, with gold as the metal. Note that at any impedance, the circuit quality factor Q_0 increases linearly with the substrate thickness h, and the radiation factor Q_r decreases as h^2. From (5) the total quality factor Q_t may thus be determined by Q_0 when the substrate is thin (Q_0 is then small) or by Q_r when the substrate is thick (Q_r is small relative to Q_0). The largest values of Q_t occur when Q_0 dominates at high Z_0 and Q_r, at low Z_0, and Q_t becomes asymptotic to the Q_0 and Q_r curves in high and low Z_0 regions. Similar curves have been obtained at different frequencies and also for different substrate dielectric constants.

The total quality factor Q_t in Fig. 1 is plotted in Fig. 2

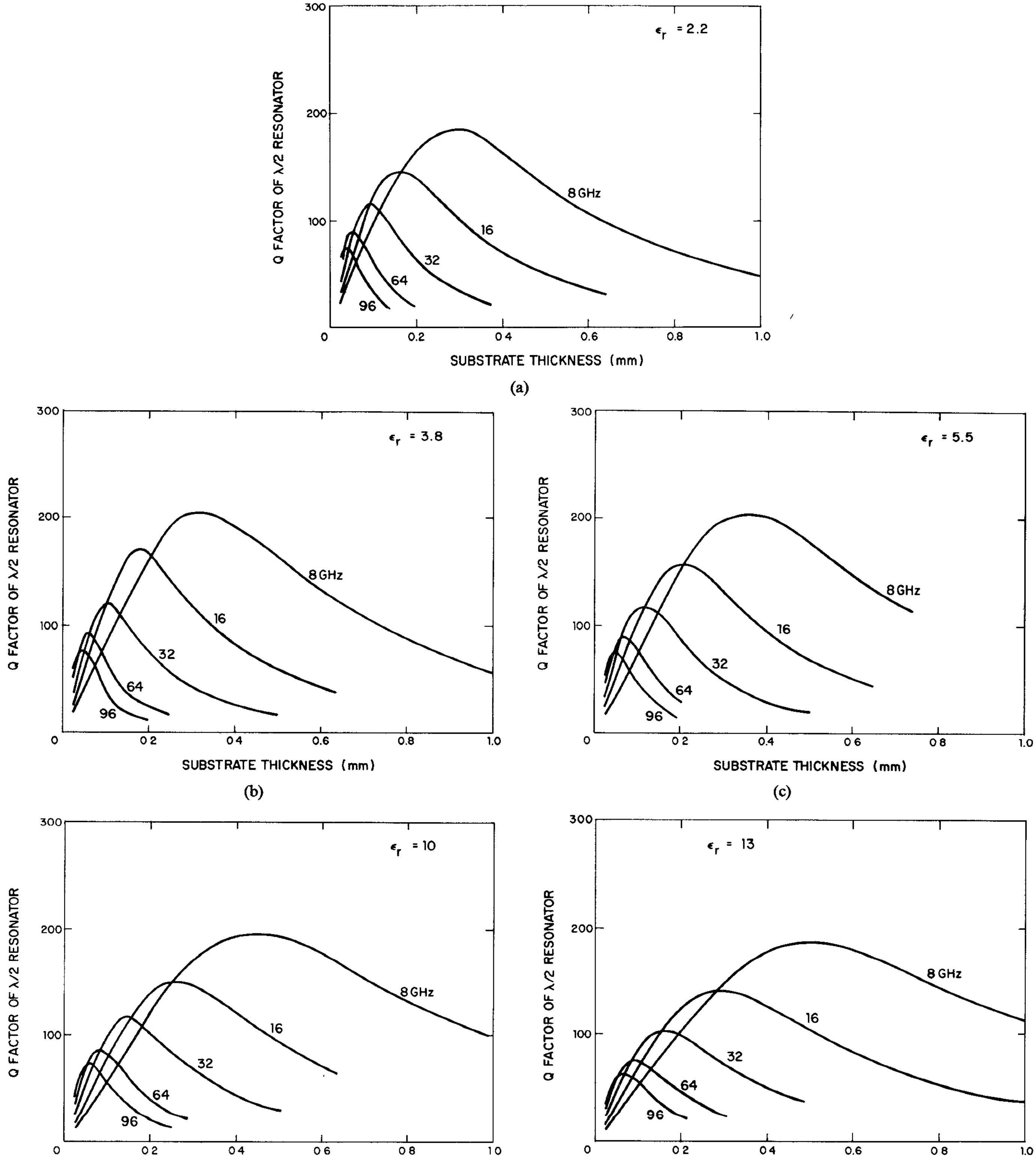

Fig. 3. Q-factor of $\lambda/2$ resonators against substrate thickness in millimeters for $Z_0=50\ \Omega$, for different frequencies. Strip metal gold 3 μm thick. Cases considered as follows. (a) $\epsilon_r=2.2$, $\tan\delta=8\times10^{-4}$. (b) $\epsilon_r=3.8$, $\tan\delta=1\times10^{-4}$. (c) $\epsilon_r=5.5$, $\tan\delta=1\times10^{-4}$. (d) $\epsilon_r=10$, $\tan\delta=1\times10^{-4}$. (e) $\epsilon_r=13$, $\tan\delta=2\times10^{-4}$ up to 16 GHz; $\tan\delta=6\times10^{-4}$ at 32 Hz; $\tan\delta=1\times10^{-3}$ at 64 GHz; $\tan\delta=1.5\times10^{-3}$ at 96 GHz.

as a function of substrate thickness for different line impedances, for $\epsilon_r=13$, $f=32$ GHz. At each impedance the curve peaks to a maximum value of Q_t at some substrate thickness. Note that this maximum Q_t decreases with increasing impedance, and furthermore, the substrate thickness at which the maximum Q_t occurs increases from

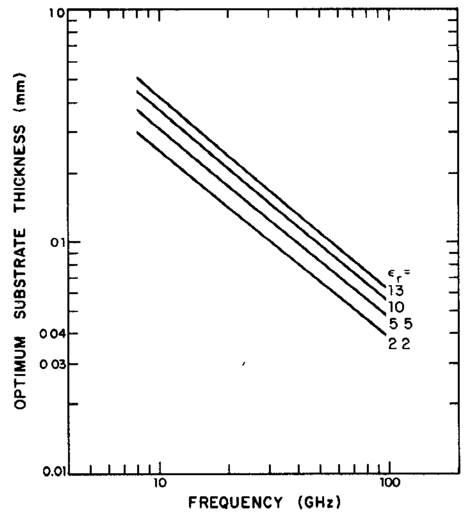

Fig. 4. Substrate thickness for maximum Q-factor against frequency for 50-Ω $\lambda/2$ resonators, for different ϵ_r.

0.11 mm for 25-Ω line to 0.225 mm for 75-Ω lines. The shape of these curves is discussed below in relation to the results obtained for different frequencies, and substrate dielectric constants in Fig. 3.

Fig. 3 plots the total quality factor Q_t against substrate thickness for 50-Ω microstrip at different frequencies. When the circuit quality factor Q_0 dominates, note that α_c varies as $\sqrt{f}$, λ_g as $1/f$ and Q_0 (and hence Q_t) varies as $\sqrt{f}$; this behavior is seen at the low values of h. For the thicker substrates, where Q_r is dominant, the variation of Q_t is $1/f^2$ (see (10)). Between these extremes, Q_t rises to a maximum at some substrate thickness depending on the frequency. The circuit quality factor Q_0 may be obtained from Fig. 3 by extrapolating the Q_t results at small h when radiation may be neglected.

The substrate thickness values corresponding to the maximum Q_t are plotted as a function of frequency in Fig. 4, for different ϵ_r. These curves show, for lower ϵ_r material, that thinner substrates yield the maximum Q_t values. The lower ϵ_r substrates have larger w/h ratios for 50-Ω line impedance and therefore smaller substrate thickness may be used to retain Q_0 at some nominal high value. The corresponding radiation quality factor Q_r increases as the substrate thickness decreases and therefore the total quality factor Q_t shows some slight increase.

Note that for wide substrates, the surface wave TE_1 cutoff [8] at any particular frequency f occurs for a substrate thickness of

$$h = \frac{c_0}{4f\sqrt{\epsilon_r - 1}} \tag{11}$$

where c_0 is the free space velocity of light. This determines the upper limit of the permissible substrate thickness, which is much larger than the optimum substrate thicknesses obtained above.

IV. Conclusions

The optimum substrate thickness as a function of frequency, for maximum Q_t of 50-Ω microstrip half-wavelength resonators, has been predicted for different ϵ_r. It appears that some slight increase in the quality factor is obtained by using lower ϵ_r substrates, $\tan\delta$ being equal, with resultant larger guide wavelengths, which may be advantageous at higher frequencies.

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